PHASE TRANSITION AND 1/*N* EXPANSION IN (2 + 1)-DIMENSIONAL SUPERSYMMETRIC SIGMA MODELS

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ABSTRACT. Three-dimensional supersymmetric generalized non-linear sigma models are shown to exhibit second-order phase transition due to spontaneous breakdown of the internal symmetry below a critical value of the coupling constant. Supersymmetry remains unbroken in both phases. Supergraph diagram technique of the corresponding $1/N$ expansion and the particle spectrum are derived.

1. Generalized non-linear sigma models (GNLSMs) in two dimensions [1] have, in the last few years, attracted a lot of interest because of their deep analogies with the realistic four-dimensional gauge theories (instantons, asymptotic freedom, confinement). In Reference [2] supersymmetric extensions of the former were proposed to naturally include interactions with fermions. The accessibility of the powerful $1/N$ expansion to the GNLSMs (for a recent review, see Reference [3]) makes them a perfect instrument to imitate the complicated structure of quantum chromodynamics.

GNLSMs in higher (Euclidean) space-time dimensions $D > 2$ are also interesting, especially in the context of phase transitions and critical phenomena [4, 5]. From this point of view we consider in the present note the supersymmetric version of $D = 3$ GNLSMs, develop their $1/N$ expansion in an explicitly supersymmetric formulation, find a second-order phase transition, and analyze the properties of the arising phases.

2. We shall discuss for concreteness $D = 3$ supersymmetric GNLSMs on real Grassmannians $G_{N,n}(R) = 0(N)/0(N-n) \times 0(n)$. The complex and quaternionic cases can be treated analogously. We shall work in the framework of the superspace approach to supersymmetry [6]. The models under consideration are described by the superspace Lagrangian density (cf. Reference [2]):

$$\mathscr{L}(x, \theta) = N\mu/T \left\{ \tfrac{1}{4} (\overline{\nabla}_\alpha \Phi)^k_a (\nabla_\alpha \Phi)^k_a - \tfrac{1}{2} \Sigma_0 (\Phi^k_a \Phi^k_a - n) - \right.$$

$$\left. - \tfrac{1}{2} \Sigma_{kl} (\Phi^k_a \Phi^l_a - 1/n\, \delta^{kl} (\Phi^r_a \Phi^r_a)) \right\}; \tag{1}$$

$$(\nabla_\alpha \Phi)^k_a = \mathscr{D}_\alpha \Phi^k_a + \mathscr{A}^{kl}_\alpha \Phi^l_a, \quad \Sigma_{kl} = \Sigma_{lk}, \ \delta^{kl} \Sigma_{kl} = 0.$$

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Here and below the following set of notations is used:

$$\Phi_a^k(x,\theta) = \varphi_a^k(x) + \bar\theta\,\psi_a^k(x) + \tfrac{1}{2}\,\bar\theta\theta\,F_a^k(x)\,,$$

$$\Sigma_0(x,\theta) = \sigma_0(x) + \bar\theta\,\kappa_0(x) + \tfrac{1}{2}\,\bar\theta\theta\,\alpha_0(x)\,,$$

$$\Sigma_{kl}(x,\theta) = \sigma_{kl}(x) + \bar\theta\,\kappa_{kl}(x) + \tfrac{1}{2}\,\bar\theta\theta\,\alpha_{kl}(x)\,, \tag{2}$$

$$\mathscr{A}_\alpha^{kl}(x,\theta) = B_\alpha^{kl}(x) - i(\not{A}^{kl}(x)\theta)_\alpha + \rho^{kl}(x)\theta_\alpha + \tfrac{1}{2}\,\theta\theta\,C_\alpha^{kl}(x), \quad \not{A}^{kl} \equiv A_\mu^{kl}\gamma^\mu;$$

$$\mathscr{D}_\alpha = \partial/\partial\bar\theta^\alpha - i(\not\partial\theta)_\alpha, \quad \not\partial \equiv \gamma^\mu\partial/\partial x^\mu.$$

The two-component Majorana spinors θ_α, $\psi^a_{\alpha,k}$, $\kappa_{\alpha,kl}$, $\kappa_{\alpha,0}$, $\mathscr{A}_\alpha^{kl}$, B_α^{kl}, C_α^{kl} (i.e., $\bar\theta_\alpha = C_{\alpha\beta}^{-1}\theta_\beta$ etc. for Dirac conjugation, $C_{\alpha\beta}$ being the charge conjugation matrix) are odd elements of a Grassmann algebra. In Equation (1) T denotes a dimensionless coupling constant (temperature) and μ is a mass scale parameter. Summation over repeated indices is understood. Indices will often be suppressed for brevity. The full symmetry group $0(N)$ acts on the 'isotopic' indices $a = 1,\dots,N$ of the scalar superfield Φ_a^k, and the local gauge group $0(n)$ acts on the 'color' indices $k, l = 1,\dots,n$ of Φ_a^k, Σ_{kl}, $\mathscr{A}_\alpha^{kl}$. Σ_0 and Σ_{kl} in (1) are auxiliary scalar superfields playing the role of Lagrange multipliers for the constraints $\Phi_a^k\Phi_a^l - \delta^{kl} = 0$ which parametrize $G_{N,n}(R)$ modulo local $0(n)$ rotations. The corresponding gauge superfield $\mathscr{A}_\alpha^{kl}$ is left, in the sequel, an independent dynamical variable, although the classical equations of motion render it a function of

$$\Phi_a^k : \mathscr{A}_\alpha^{kl} = \tfrac{1}{2}\,\Phi_a^k\,\overleftrightarrow{\mathscr{D}}_\alpha\Phi_a^l.$$

Because of the $0(n)$ superfield gauge invariance of $\mathscr{L}(1)$:

$$\Phi_a^k(x,\theta) \longrightarrow \Omega_{kl}^{-1}(x,\theta)\,\Phi_a^l(x,\theta), \quad \Omega(x,\theta) \in 0(n),$$

$$\mathscr{A}_\alpha(x,\theta) \longrightarrow \Omega^{-1}(x,\theta)\,\mathscr{A}_\alpha(x,\theta)\,\Omega(x,\theta) + \Omega^{-1}(x,\theta)\,\mathscr{D}_\alpha\Omega(x,\theta),$$

we must introduce according to the Faddeev–Popov procedure [7] an additional gauge fixing term and the associated superfield ghosts:

$$\Delta\mathscr{L}(x,\theta) = N/\lambda(\bar{\mathscr{D}}_\alpha\mathscr{A}_\alpha)^2 + \chi_k^*\,\bar{\mathscr{D}}_\alpha\nabla_\alpha^{kl}\chi_l\,. \tag{3}$$

3. To derive the $1/N$ expansion of the quantum generating functional:

$$Z[J, L_0, L, K] = \int \Pi \, d\Phi \, d\Sigma_0 \, d\Sigma \, d\mathscr{A} \, d\chi \, d\chi^* \exp \{ i \int d^3x \, d^2\theta \times$$

$$\times [\mathscr{L}(x, \theta) + J_a^k \Phi_a^k + L_0 \Sigma_0 + L_{kl}\Sigma_{kl} + K_\alpha^{kl} \mathscr{A}_\alpha^{kl}] \} \quad (4)$$

by the saddle point method the following basic formulas for superfield Gaussian functional integration are to be used:

$$\int \Pi \, d\Phi \exp \{ -\tfrac{1}{2} \int d^3x \, d^2\theta \, d^3y \, d^2\theta' \Phi(x, \theta) M(x, \theta; y, \theta') \Phi(y, \theta') +$$

$$+ \int d^3x \, d^2\theta \, J(x, \theta)\Phi(x, \theta)\} = (\mathrm{Ber}\, M)^{-1/2} \exp \{ \tfrac{1}{2} \int d^3x \, d^2\theta \, d^3y \, d^2\theta' \times \quad (5)$$

$$\times J(x, \theta) M^{-1}(x, \theta; y, \theta') J(y, \theta')\}; \qquad \mathrm{Ber}\, M = \exp \{ \mathrm{sTr} \log M\},$$

where Ber (.) and sTr (.) denote Berezinian (superspace analogue of the determinant) and supertrace respectively (for a comprehensive recent review of the mathematical theory of superalgebras and supermanifolds, see Reference [8]). Equations (5) can be easily proved by means of the component decomposition (2) and the rules for integration over Grassmann variables [9].

Let us split Φ_a^k into (cf. [5]):

$$(\Phi_a^k) = (U_{a_1}^k, V_{a_2}^k) \qquad a_1 = 1, \ldots, n, \; a_2 = n + 1, \ldots, N$$

and perform the Gaussian integration over $V_{a_2}^k$ in (4) according to Equations (5):

$$Z[J, L_0, L, K] = \int \Pi \, dU \, d\Sigma_0 \, d\Sigma \, d\mathscr{A} \, d\chi \, d\chi^* \exp \left\{ iNS_1 [U, \Sigma_0, \Sigma, \mathscr{A}] + \right.$$

$$\left. + iS_2 [U, \Sigma_0, \Sigma, \mathscr{A}, \chi, \chi^*; J, L_0, L, K] \right\}; \quad (6)$$

$$S_1 \equiv i/2 \; \mathrm{sTr} \log [\tfrac{1}{2} (\bar\nabla \nabla) + \Sigma_0 + \Sigma] + \mu/\mathrm{T} \int d^3x \, d^2\theta \, \{ \tfrac{1}{4} (\bar\nabla U)_{a_1}^k (\nabla U)_{a_1}^k -$$

$$- \tfrac{1}{2} \Sigma_0 (U_{a_1}^k U_{a_1}^k - n) - \tfrac{1}{2} \Sigma_{kl} (U_{a_1}^k U_{a_1}^l - 1/n \, \delta^{kl} (U_{a_1}^r U_{a_1}^r))\}; \quad (7)$$

$$S_2 \equiv in/2 \; \mathrm{sTr} \log [\tfrac{1}{2} (\bar\nabla \nabla) + \Sigma_0 + \Sigma] + \int d^3x \, d^2\theta \times$$

$$\times [jU + L_0 \Sigma_0 + L\Sigma + K_\alpha \mathscr{A}_\alpha + \Delta\mathscr{L}(x, \theta)] + T/2N\mu \int d^3x \, d^2\theta \, d^3y \, d^2\theta' \times$$

$$\times \{ h(\tfrac{1}{2} \bar\nabla \nabla + \Sigma_0 + \Sigma)^{-1} h\}; \; (J_a^k) \equiv (j_{a_1}^k, h_{a_2}^k).$$

The $1/N$ expansions of (6) is generated through expansion around the constant saddle points:

$$\hat{U}^k_{a_1} = \hat{\varphi}^k_{a_1} + \delta(\theta)\hat{F}^k_{a_1}, \qquad \hat{\Sigma}_0 = \sigma_0 + \delta(\theta)\hat{\alpha}_0, \qquad \hat{\Sigma}_{kl} = \hat{\sigma}_{kl} + \delta(\theta)\hat{\alpha}_{kl} \qquad \hat{\mathscr{A}}^{kl}_{\alpha} = 0 \tag{8}$$

of S_1 (7) ($\delta(\theta) = \frac{1}{2}\bar{\theta}\theta$ being the Grassmann delta-function). The equations for determination of (8) read:

$$(\hat{\sigma}_0 + \hat{\sigma})\hat{F} + (\hat{\alpha}_0 + \hat{\alpha})\hat{\varphi} = 0; \qquad (\hat{\sigma}_0 + \hat{\sigma})\hat{\varphi} - \hat{F} = 0;$$

$$\mu/2T\,(\hat{\varphi}^k_{a_1}\hat{F}^l_{a_1} + \hat{\varphi}^l_{a_1}\hat{F}^k_{a_1}) - i\int\frac{d^3p}{(2\pi)^3}\,\{(\hat{\sigma}_0 + \hat{\sigma})_{kr}[\hat{\alpha}_0 + \hat{\alpha} + (\hat{\sigma}_0 + \hat{\sigma})^2 - p^2]^{-1}_{rl} +$$

$$+ [\hat{\alpha}_0 + \hat{\alpha} + (\hat{\sigma}_0 + \hat{\sigma})^2 - p^2]^{-1}_{kr}(\hat{\sigma}_0 + \hat{\sigma})_{rl} - 2(\hat{\sigma}_0 + \hat{\sigma})_{kr}[(\hat{\sigma}_0 + \hat{\sigma})^2 - p^2]^{-1}_{rl}\} = 0; \tag{9}$$

$$\mu/T\,(\hat{\varphi}^k_{a_1}\hat{\varphi}^l_{a_1} - \delta^{kl}) - i\int\frac{d^3p}{(2\pi)^3}\,[\hat{\alpha}_0 + \hat{\alpha} + (\hat{\sigma}_0 + \hat{\sigma})^2 - p^2]^{-1}_{kl} = 0.$$

It is straightforward to deduce that the only possible solution of Equations (9) for $\hat{\Sigma}_{kl}$ is $\hat{\Sigma}_{kl} = 0$. The divergent integral in the last Equation (9) can be renormalized by, e.g., a 'soft mass' subtraction:

$$-i\int\frac{d^3p}{(2\pi)^3}\,\{[\hat{\alpha}_0 + \hat{\sigma}_0^2 - p^2]^{-1}[\mu^2 - p^2]^{-1}\} + a_0/4\pi = \mu/T_c - 1/4\pi(\hat{\alpha}_0 + \hat{\sigma}_0^2)^{1/2}$$

$$T_c \equiv 4\pi(1 + a_0)^{-1},$$

a_0 being an arbitrary dimensionless constant accounting for the subtraction ambiguity. Now solving the system (9) with $\hat{\sigma}_{kl} = \hat{\alpha}_{kl} = 0$ we obtain the following set of solutions:

(i) High temperature (HT) phase solutions ($T > T_c$):

(HT$_1$) $\hat{\varphi}^k_{a_1} = \hat{F}^k_{a_1} = \hat{\alpha}_0 = 0, \qquad \hat{\sigma}_0 = 4\pi\mu(1/T_c - 1/T);$

(HT$_2$) $\hat{\varphi}^k_{a_1} = \hat{F}^k_{a_1} = \hat{\sigma}_0 = 0, \qquad \hat{\alpha}_0^{1/2} = 4\pi\mu(1/T_c - 1/T);$

(ii) Low temperature (LT) phase solutions ($T < T_c$):

(LT$_1$) $\hat{\varphi}^k_{a_1}\hat{\varphi}^l_{a_1} = (1 - T/T_c)\delta^{kl}, \qquad \hat{F}^k_{a_1} = \hat{\sigma}_0 = \hat{\alpha}_0 = 0;$

(LT$_2$) $\hat{\varphi}^k_{a_1}\hat{\varphi}^l_{a_1} = (1 - T/T_c)\delta^{kl}, \qquad \hat{\alpha}_0^{1/2} = -\hat{\sigma}_0 = 4\pi\mu(1/T - 1/T_c),\ \hat{F}^k_{a_1} = \hat{\sigma}_0\hat{\varphi}^k_{a_1};$

(iii) Critical theory ($T = T_c$): $\hat{\varphi}^k_{a_1} = \hat{F}^k_{a_1} = \hat{\sigma}_0 = \hat{\alpha}_0 = 0.$

Consider the quantum effective potential $\mathscr{V}[\hat{U}, \hat{\Sigma}_0, \hat{\Sigma}, \hat{\mathscr{A}}]$ of (1) which according to the general theory is given by means of the Legendre transform of $\log Z[J, L_0, L, K]$ with respect to constant J, L_0, L, K (see, e.g., [10]). Noticing that the factor N enters the exponent of the generating functional (6) like $\hbar^{-1}$ ($\hbar$ being the Planck constant) we have in the limit $N \to \infty$:

$$\mathrm{Vol}\ \mathscr{V}_\infty[\hat{U}, \hat{\Sigma}_0, \hat{\Sigma}, \hat{\mathscr{A}}] \equiv \mathrm{Vol} \lim_{N\to\infty} \mathscr{V}[\hat{U}, \hat{\Sigma}_0, \hat{\Sigma}, \hat{\mathscr{A}}] = -S_1[\hat{U}, \hat{\Sigma}_0, \hat{\Sigma}, \hat{\mathscr{A}}], \quad \mathrm{Vol} \equiv \int d^3x.$$

From this equation one deduces that the saddle points (8), (i) – (iii) of S_1 are vacuum expectation values of the corresponding superfields in the leading $1/N$ order:

$$\langle \Phi(x, \theta) \rangle = \hat{U} + 0(1/N), \qquad \langle \Sigma_{0,kl}(x, \theta) \rangle = \hat{\Sigma}_{0,kl} + 0(1/N), \qquad \langle \mathscr{A}^{kl}_\alpha(x, \theta) \rangle = 0,$$

and, therefore, to describe the real vacua (i.e., the pure phases) $\hat{U}, \hat{\Sigma}_0, \hat{\Sigma}_{kl}$ must realize absolute minima of $\mathscr{V}_\infty$. Evaluating $\mathscr{V}_\infty$ from equation (7) by a straightforward computation:

$$\mathscr{V}_\infty[\hat{U}, \hat{\Sigma}_0, \hat{\Sigma}, = 0, \hat{\mathscr{A}} = 0] = \mu/T\ \{\hat{\sigma}_0 \hat{\varphi}^k_{a_1} \hat{F}^k_{a_1} - \tfrac{1}{2} \hat{F}^k_{a_1} \hat{F}^k_{a_1} +$$

$$+ \tfrac{1}{2}[\hat{\varphi}^k_{a_1} \hat{\varphi}^k_{a_1} - n(1 - T/T_c)]\} + n/12\pi[\hat{\sigma}_0^3 - (\hat{\alpha}_0 + \hat{\sigma}_0^2)^{3/2}] \tag{10}$$

and substituting in Equation (10) the $(\mathrm{HT}_{1,2})$ and $(\mathrm{LT}_{1,2})$ solutions we get:

$$\mathscr{V}_\infty\Big|_{\mathrm{HT}_1} = \mathscr{V}_\infty\Big|_{\mathrm{LT}_1} = 0; \qquad \mathscr{V}_\infty\Big|_{\mathrm{HT}_2} = \mathscr{V}_\infty\Big|_{\mathrm{LT}_2} = n/24\pi\, \hat{\alpha}_0^{3/2} > 0.$$

Thus only (HT_1) and (LT_1) phases are realized. An important consequence of this is that supersymmetry is preserved in both HT and LT phases as well as at the critical point (the corresponding non-zero vacuum expectation values $\langle \Sigma_0 \rangle = \hat{\sigma}_0$, $\langle \Phi^k_a \rangle = \hat{\varphi}^k_{a_1}$ respectively, do not violate supersymmetry).

4. It is convenient in what follows to rescale $\Phi \to (\mu/T)^{1/2}\Phi$, $J \to (T/\mu)^{1/2} J$. Shifting $U = \hat{\varphi} + N^{-1/2}\tilde{U}$, $\Sigma_0 = \hat{\sigma}_0 + \tilde{\Sigma}_0$ in Equation (6) and performing the Gaussian integration over $\tilde{U}$ we arrive at the expression:

$$Z[J, L_0, L, K] = \int \Pi\, d\tilde{\Sigma}_0\, d\Sigma\, d\mathscr{A}\, d\chi\, d\chi^*\, \exp[-N/4 \int d^3x\, d^2\theta\, \{\bar{\mathscr{A}}\mathscr{A}\, G(x, \theta; x, \theta) -$$

$$-i(\bar{\mathscr{A}}\hat{\varphi})(\mathscr{A}\hat{\varphi})\} + N/2 \sum_{M=2}^{\infty} (-1)^M/_M\, \mathrm{sTr}\,[(GW)^M] + i/2 \int d^3x\, d^2\theta\, d^3y\, d^2\theta' \times$$

$$\times \{j - (\widetilde{\Sigma}_0 + \Sigma + \tfrac{1}{2} \mathscr{A}_\alpha \overline{\nabla}_\alpha)\hat{\varphi} N^{1/2}] (x,\theta)(G^{-1}+W)^{-1}(x,\theta;y,\theta') \times$$

$$\times [j - (\widetilde{\Sigma}_0 + \Sigma - \overleftarrow{\overline{\nabla}}_\alpha \mathscr{A}_\alpha)\hat{\varphi} N^{1/2}] (y,\theta') + h(G^{-1}+W)^{-1}h\} + i \int d^3x\, d^2\theta \times$$

$$\times \{L_0\Sigma_0 + L\Sigma + K_\alpha \mathscr{A}_\alpha + \Delta\mathscr{L}(x,\theta)\}] ; \qquad (11)$$

$$G \equiv [\tfrac{1}{2} \overline{\mathscr{D}}\mathscr{D} + \hat{\sigma}_0]^{-1}; \qquad W \equiv \tfrac{1}{2}(\overline{\mathscr{A}}\mathscr{D} - \overleftarrow{\overline{\mathscr{D}}}\mathscr{A} + \overline{\mathscr{A}}\mathscr{A}) + \widetilde{\Sigma}_0 + \Sigma .$$

Equation (11) determines all supergraph elements of the $1/N$ diagram expansion in the HT and LT phases and in the critical theory. Let us write down explicitly the free propagators in momentum space choosing Lorentz-type gauge $\lambda = 0$ in $\Delta\mathscr{L}$ (3) (i.e., $\overline{\mathscr{D}}_\alpha \mathscr{A}_\alpha = 0$):

$$\langle \Phi^k_a \Phi^l_b \rangle^{(0)}_{\mathrm{HT}} = (-i)\, e^{\bar{\theta} \not{p} \theta'} (1 + \sigma_0 \delta(\theta-\theta'))\, [\hat{\sigma}_0^2 - p^2]^{-1} \delta^{kl}\delta_{ab}; \qquad (12.a)$$

$$\langle \widetilde{\Sigma}_0 \widetilde{\Sigma}_0 \rangle^{(0)}_{\mathrm{HT}} = -2i(Nn)^{-1} [(4\hat{\sigma}_0^2 - p^2)F(p^2)]^{-1} e^{\bar{\theta} \not{p} \theta'} (1 - 2\hat{\sigma}_0 \delta(\theta-\theta')); \qquad (12.b)$$

$$\langle \Sigma_{kl}\Sigma_{rs} \rangle^{(0)}_{\mathrm{HT}} = n \langle \widetilde{\Sigma}_0 \widetilde{\Sigma}_0 \rangle^{(0)}_{\mathrm{HT}} \{\tfrac{1}{2}(\delta_{kr}\delta_{ls} + \delta_{ks}\delta_{lr}) - 1/n\, \delta_{kl}\delta_{rs}\}; \qquad (12.c)$$

$$\langle \mathscr{A}^{kl}_\alpha \overline{\mathscr{A}}^{rs}_\beta \rangle^{(0)}_{\mathrm{HT}} = 2iN^{-1} [\not{p}(2\hat{\sigma}_0 - \not{p})F(p^2)]^{-1} (\delta^{kr}\delta^{ls}\delta^{ks}\delta^{lr})\, \Pi_{\alpha\beta}, \qquad (12.d)$$

$$\Pi(p;\theta,\theta') = \tfrac{1}{2}\, e^{\bar{\theta} \not{p} \theta'} [\delta(\theta-\theta') - 1/\not{p}], \quad \not{p} \equiv p_\mu \gamma^\mu, \qquad (13)$$

$$F(p^2) = \begin{cases} [4\pi(-p^2)^{1/2}]^{-1}\, \mathrm{tg}^{-1} [(-p^2/4\hat{\sigma}_0^2)^{1/2}], & p^2 < 0, \\ [4\pi(p^2)^{1/2}]^{-1} \log [2\hat{\sigma}_0 + (p^2)^{1/2})(4\hat{\sigma}_0^2 - p^2)^{1/2}], & p^2 > 0; \end{cases} \qquad (14)$$

$$\langle V^k_{a_2} V^l_{b_2} \rangle^{(0)}_{\mathrm{LT}} = \delta^{kl}\delta_{a_2 b_2}\, i(p^2)^{-1} e^{\bar{\theta} \not{p} \theta'}; \qquad (15.a)$$

$$\langle \Phi^k_r \Phi^l_s \rangle^{(0)}_{\mathrm{LT}} = \tfrac{1}{2}(\delta^{kl}\delta_{rs} - \delta_{ks}\delta_{lr})\, i(p^2)^{-1} e^{\bar{\theta} \not{p} \theta'} - i\, e^{\bar{\theta} \not{p} \theta'} [-p^2 + 16\hat{\varphi}^2(-p^2)^{1/2}]^{-1} \times$$

$$\times \{\tfrac{1}{2}(\delta^{kl}\delta_{rs} + \delta_{ks}\delta_{rl}) - (1 - 1/n)\delta_{kr}\delta_{ls} 16\hat{\varphi}^2 [(-p^2)^{1/2} + 16n\hat{\varphi}^2]^{-1}\}, \qquad (15.b)$$

$$\Phi^k_r \equiv (\hat{\varphi}^2)^{-1} \hat{\varphi}^r_{a_1} \tilde{U}^k_{a_1}, \qquad \hat{\varphi}^2 \delta^{kl} = \hat{\varphi}^k_{a_1} \hat{\varphi}^l_{a_1} \qquad \text{(cf. (ii))};$$

$$\langle \tilde{\Sigma}_0 \tilde{\Sigma}_0 \rangle^{(0)}_{\mathrm{LT}} = -16i(Nn)^{-1} [(-p^2)^{1/2} + 16 \hat{\varphi}^2]^{-1} e^{\theta \not{p} \theta'}; \tag{15.c}$$

$$\langle \Sigma_{kl} \Sigma_{rs} \rangle^{(0)}_{\mathrm{LT}} = -16iN^{-1} [(-p^2)^{1/2} + 16\hat{\varphi}^2]^{-1} e^{\bar{\theta} \not{p} \theta'} \times$$
$$\times \{ \tfrac{1}{2} (\delta_{kr}\delta_{ls} + \delta_{ks}\delta_{lr}) - 1/n\, \delta_{kl}\delta_{rs} \}; \tag{15.d}$$

$$\langle \mathscr{A}^{kl}_\alpha \mathscr{A}^{rs}_\beta \rangle^{(0)}_{\mathrm{LT}} = 16iN^{-1} [(-p^2)^{1/2} + 16\varphi^2]^{-1} (\delta^{kr}\delta^{ls}\delta^{ks}\delta^{lr}) \Pi_{\alpha\beta}(p; \theta, \theta'). \tag{15.e}$$

The operator $\Pi_{\alpha\beta}$ (13) is a 'transverse' projector: $\bar{\mathscr{D}}_\alpha \Pi_{\alpha\beta} = 0$, $\Pi_{\alpha\gamma}\Pi_{\gamma\beta} = \Pi_{\alpha\beta}$ ($\Pi_{\alpha\beta}$ is the supersymmetric analogue of the transverse projector $\Pi_{\mu\nu}(p) = g_{\mu\nu} - p_\mu p_\nu / p^2$ in the gauge field propagators of the usual GNLSMs [4, 5]). From formulas (12), (15) we read off directly the particle spectrum in the corresponding phases.

The (HT_1) phase is super-, $0(N)$ isotopic-, and $0(n)$ gauge-symmetric. There are nN massive scalar superfields Φ^k_a with equal mass $m \equiv \hat{\sigma}_0$ which is dynamically generated (it was not present in the initial classical Lagrangian (1)) and $n(n-1)$ massless transverse gauge superfields $\mathscr{A}^{kl}_\alpha$ (12d). The singularities of (12b, c, d) at $p^2 = 4\hat{\sigma}_0^2$ do not correspond to any (bound state) particle because of the presence of an additional logarithmic singularity in $F(p^2)$ (14) at this point.

The (LT_1) phase (Higgs–Goldstone phase) is supersymmetric with spontaneous breaking of the $0(N) \times 0(n)_{\mathrm{gauge}}$ symmetry up to the residual $0(N-n) \times \mathrm{diag}\,(0(n)_{\mathrm{isotopic}} \times 0(n)_{\mathrm{gauge}})$ symmetry. This last subgroup is the stability group of the LT vacuum defined through $\hat{\varphi}^k_{a_1}$ in (ii, LT_1). The LT particle spectrum consists of only $n(N-1)$ massless Goldstone superfields $V^k_{a_2}$, Φ^k_r $(r \neq k)$ (15a, b). The propagators of Φ^k_k (no summation over k), $\tilde{\Sigma}_0$, Σ_{kl} and $\mathscr{A}^{kl}_\alpha$ (15b-e) have not poles at all. Thus, the Higgs mechanism does not operate here. Let us stress on the similarities of the HT and LT phases in the $D = 3$ supersymmetric GNLSMs to their counterparts in the usual non-supersymmetric case, see References [4, 5]. (The description of the LT particle spectrum in the usual GNLSMs ($D = 3$) given in Reference [5] is not correct. The former contains only $n(N-1)$ massless Goldstone bosons, the remaining n scalar bosons and the would-be massive (via Higgs mechanism) gauge bosons being 'confined', just as in the supersymmetric case.)

The critical theory (iii) with superfield propagators:

$$\langle \Phi^k_a \Phi^l_b \rangle^{(0)}_{cr} = \delta^{kl}\delta_{ab}\, i(p^2)^{-1} e^{\bar{\theta} \not{p} \theta'}, \qquad \langle \tilde{\Sigma}_0 \tilde{\Sigma}_0 \rangle^{(0)}_{cr} = -16i(Nn)^{-1} (-p^2)^{1/2} e^{\bar{\theta} \not{p} \theta'},$$

$$\langle \Sigma_{kl} \Sigma_{rs} \rangle^{(0)}_{cr} = n \langle \tilde{\Sigma}_0 \tilde{\Sigma}_0 \rangle^{(0)}_{cr} \{ \tfrac{1}{2} (\delta_{kr}\delta_{ls} + \delta_{ks}\delta_{lr}) - 1/n\, \delta_{kl}\delta_{rs} \},$$

$$\langle \mathscr{A}^{kl}_\alpha \bar{\mathscr{A}}^{rs}_\beta \rangle^{(0)}_{cr} = 16iN^{-1} (-p^2)^{1/2} (\delta^{kr}\delta^{ls}\delta^{ks}\delta^{lr}) \Pi_{\alpha\beta}(p; \theta, \theta'),$$

is supersymmetric and scale invariant.

In a subsequent work the problems of renormalization of the $1/N$ expansion and the critical behaviour will be considered.